EMISSIVITY OF THREE-DIMENSIONAL BODIES AND ANGULAR COEFFICIENTS IN REAL-MEDIUM SYSTEMS

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A new method is proposed for calculating the emissivity of three-dimensional bodies and the angular coefficients in real-medium systems. Use is made of published formulas for gray bodies and of one-dimensional nomograms constructed on the basis of experimental data.

There is no basic difficulty in accounting for the geometry of a body in idealized gray-medium systems, although a digital computer is often used. Many articles have been written on this subject. Most of the results are of no value in problems involving a real medium, because of the wide differences between real and gray radiation characteristics. Hence those methods and typical examples where developments in the theory of gray systems find wide application have become of greater interest.

All formulas for the emissivity, the angular coefficients, and derivative quantities can, in the case of monochromatic (or gray) radiation, be reduced to the form

$$c_0 + \sum_j c_j f_j(\tau_\omega). \tag{1}$$

The coefficients c_0 and c_j do not depend on the wave number. In order to apply formulas (1) to real-medium systems, one must integrate over the spectrum. We will consider here only examples involving a thermal radiation flux. The calculation of emissivity amounts to actually calculating the thermal absorptive power of a medium at temperature $T = T_0$. For the complete spectrum (1) becomes

$$c_0 + \sum_j c_j \frac{\pi}{\sigma T^4} \int_0^\infty I_{0\omega} f_j(\tau_\omega)\, d\omega. \tag{2}$$

The proposed method of calculation is narrowed down by still another condition. All functions f_j must be approximated by the expression

$$f_j(\tau_\omega) = \sum_i a_i \exp(-b_i \tau_\omega). \tag{3}$$

Transforming this and inserting into (2) will yield

$$c_0 + \sum_j c_j \left\{ \sum_i a_i - \sum_i a_i \frac{\pi}{\sigma T^4} \int_0^\infty I_{0\omega} [1 - \exp(-b_i \alpha_\omega x)]\, d\omega \right\}.$$

Here $\frac{\pi}{\sigma T^4} \int_0^\infty I_{0\omega}[1 - \exp(-b_i \alpha_\omega x)] d\omega = \varepsilon(b_i x)$ is the one-dimensional emissivity of a $b_i x$-long segment of the medium at temperature T. Finally, instead of (2), we have

$$c_0 + \sum_j c_j \left[\sum_i a_i - \sum_i a_i \varepsilon(T,\ b_i x) \right]. \tag{4}$$

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The ε-values are read off known nomograms. The coefficients c_0, c_j, a_i, and b_i are given by formulas derived for gray bodies. In this way, formula (4) can in many cases be used as it stands. This will be illustrated on examples given by Mikk [1, 2, 3] and by myself [4, 5, 6, 7]. In [1, 2, 3] f_j is represented by the function E_3 and the intermediate functions M, N_1, N_2, S_2. In [4, 5] the intermediate functions are expressed in terms of special functions Ki_n. Functions E_3, Ki_n, and thus also M, N_1, N_2, S_2 in [5, 6] are approximated according to formula (3) – very accurately within certain intervals of the argument. It is favorable that function f_j can often be expressed in terms of integrals of exponential functions: in such cases formula (3) goes into a quadrature. The coefficients a_i and b_i are calculated, to the first approximation, from the nodes and the weights of the Gauss quadrature. They are then refined by iteration. Formula (4), which has been derived here, represents a new target toward which the earlier effort on approximating function f_j must be continued.

Let us proceed now to the illustrative examples – to problems with original solutions. The numerical calculations apply essentially to gaseous carbon dioxide, which has a bright and discrete spectrum. As the argument we use the product pl where l is the characteristic dimension of a body. The emissivity nomograms are used according to [8].

Example 1. Emissivity of an Infinite Strip. For monochromatic radiation

$$\varepsilon_{1\omega} = 1 - 2E_3(\alpha_\omega l).$$

In [6] the following approximation is given:

$$2E_3(\tau) = 0.0628\exp(-8.8\tau) + 0.4444\exp(-2\tau) + 0.4928\exp(-1.125\tau).$$

The error does not exceed 0.6% on the interval τ [0,2]. According to Eq. (4), we have

$$\varepsilon_1 = 0.0628\,\varepsilon(8.8l) + 0.4444\,\varepsilon(2l) + 0.4928\,\varepsilon(1.125l). \tag{5}$$

For CO_2 and H_2O the results according to (5) agree closely with those which Nevskii has obtained in [9] by a very precise quadrature.

The angular coefficient for the surfaces of a strip is $1 - a_{01}$, where for a band spectrum:

$$a_0 = (T/T_0)^m\,\varepsilon[T_0,\ x(T/T_0)^u]. \tag{6}$$

For CO_2 and H_2O the exponents m and u are used according to Hottel. Combining Eqs. (5) and (6) yields a simple expression for the angular coefficient:

$$\varphi_1 = 1 - (T/T_0)^m\{0.0628\,\varepsilon[T_0,\ 8.8l(T/T_0)^u] + 0.4444\,\varepsilon[T_0,\ 2l(T/T_0)^u] + 0.4928\,\varepsilon[T_0,\ 1.125l(T/T_0)^u]\}. \tag{7}$$

Formula (6) is used in [10] for calculating the absorptive power of a nonisothermal segment also in the case of a gray incident radiation flux. These examples extend considerably the range of validity of Eq. (4) for practical calculations.

Example 2. Local Emissivity of an Infinitely Long Semicylinder, at the Center of Its Plane Base. For monochromatic radiation

$$\varepsilon_{2\omega} = 1 - M(\tau_\omega),\ \tau_\omega = \alpha_\omega R.$$

According to [5],

$$M(\tau) = 0.046\exp(-3.4\tau) + 0.317\exp(-1.42\tau) + 0.637\exp(-1.04\tau).$$

The error does not exceed 1% for $0 \le \tau \le 5$. According to Eq. (4),

$$\varepsilon_2 = 0.046\,\varepsilon(3.4R) + 0.317\,\varepsilon(1.42R) + 0.637\,\varepsilon(1.04R). \tag{8}$$

The local angular coefficient for this point and the surface containing the volume is $1 - a_{02}$. In the case of a band spectrum, the formula for φ_2 is analogous to (7).

Example 3. Emissivity of an Infinite Cylinder. For monochromatic radiation

$$\varepsilon_{3\omega} = 1 - S_2(\alpha_\omega D).$$

According to [5],

$$S_2(\tau) = 0.035\exp(-0.206\tau) + 0.235\exp(-0.51\tau) + 0.56\exp(-1.04\tau) + 0.17\exp(-1.57\tau).$$

The error is approximately 1% for $0 \le \tau \le 5$. According to Eq. (4),

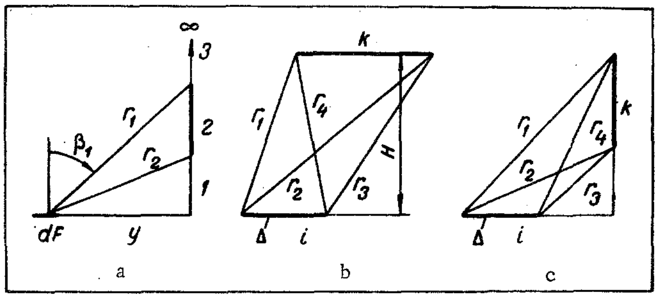

Fig. 1. Configurations for calculating the angular coefficients.

$$\varepsilon_3 = 0.035\,\varepsilon\,(0.206\,D) + 0.235\,\varepsilon\,(0.51\,D) + 0.56\,\varepsilon\,(1.04\,D) + 0.17\,\varepsilon\,(1.57\,D). \tag{9}$$

The angular coefficient for a shell "onto itself" is $1 - a_{03}$. Its calculation has been explained in Example 1.

Example 4. Two-Dimensional Local Angular Coefficient for an Element dF and a Normally Oriented Strip (Fig. 1a). The coefficient for strip 3, extending from under angle β_1 to infinity will be taken as the principal one:

$$\xi_{dF,3} = \frac{1}{2}\sin\beta_1 N_1(r_1).$$

For the semiinfinite strip ($\beta_1 = \pi/2$):

$$\xi_{dF(1+2+3)} = \frac{1}{2} N_1(y).$$

The meaning of function N_1 becomes apparent here. The distributive law applied to $\xi_{dF,3}$ yields formulas for any strip. In the case of a real medium also, it suffices to use the formula for $\xi_{dF,3}$. According to [5],

$$N_1(\tau) = 0.36\exp(-8.5\,\tau) + 0.53\exp(-1.9\,\tau) + 0.11\exp(-1.08\,\tau).$$

According to Eq. (4), we have

$$\xi'_{dF,3} = \frac{1}{2}\sin\beta_1\,[1 - 0.36\,\varepsilon\,(8.5 r_1) - 0.53\,\varepsilon\,(1.9\,r_1) - 0.11\varepsilon\,(1.08\,r_1)].$$

Here ξ' is the angular coefficient when the medium and element dF are at the same temperature. When their temperatures are not the same, we have for a band spectrum:

$$\xi_{dF,3} = \frac{1}{2}\sin\beta\left\{1 - (T/T_0)^m\left[0.36\,\varepsilon\left(T_0,\ 8.5\,r_1\frac{T_0}{T}\right)\right.\right.$$
$$\left.\left. + 0.53\,\varepsilon\left(T_0,\ 1.9\,r_1\frac{T_0}{T}\right) + 0.11\,\varepsilon\left(T_0,\ 1.08\,r_1\frac{T_0}{T}\right)\right]\right\}.$$

Example 5. Mean Angular Coefficient for Parallel Strips (Fig. 1b). According to the approximate Mikk formula derived for gray radiation,

$$\varphi_{\omega,ik} = \frac{H}{2\Delta}\left\{(\rho_2 - \rho_1 + \rho_4 - \rho_3)\,2E_3(\alpha_\omega H) + \left(\frac{1}{\rho_2} - \frac{1}{\rho_1} + \frac{1}{\rho_4} - \frac{1}{\rho_3}\right)[2E_3(\alpha_\omega H) - M(\alpha_\omega H)]\right\}.$$

The change of functions E_3 and M according to (3) and the introduction of the absorptive power will yield Eq. (4) for the angular coefficient φ_{ik}. This formula is not shown here, because it follows from the preceding examples.

In the studies made by Mikk we find many other examples where the formulas for the angular coefficient can be used as they stand in the case of real-medium bodies. They are easily written down after acquaintance with the preceding examples. We will now consider an opposite example, where the approximation of functions in the form (3) must be further refined.

Example 6. Angular Coefficient for Perpendicular Strips (Fig. 1c). According to the exact Mikk formula for a gray medium,

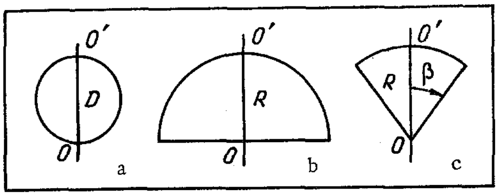

Fig. 2. Sections of two-dimensional bodies.

$$\varphi_{\omega 0}=\frac{1}{\alpha_\omega \Delta}\left[N_2(\alpha_\omega r_1)-N_2(\alpha_\omega r_2)+N_2(\alpha_\omega r_3)-N_2(\alpha_\omega r_4)\right].$$

In [5] this author has obtained an approximation of function N_2 in the form (3). This approximation is not adequate, however. In order to adapt Eq. (4), it becomes necessary to approximate the function $N_2(\alpha r)/\alpha r$. In view of the singularity as $\alpha r \to 0$, formula (3) is limited at small αr-values. This is circumvented by introducing a new function:

$$N_3(\alpha r)=[N_2(0)-N_2(\alpha r)]/\alpha r,$$

where $N_2(0) = 4\pi/3$. Function N_3 varies within the interval [0, 1]. Therefore, it can be easily approximated by Eq. (3). The angular coefficient becomes:

$$\varphi_{\omega 0}=\frac{1}{\Delta}\left[r_2N_3(\alpha_\omega r_2)-r_1N_3(\alpha_\omega r_1)+r_4N_3(\alpha_\omega r_4)-r_3N_3(\alpha_\omega r_3)\right].$$

The restriction on adapting Eq. (4) has been removed. An analogous procedure can be adopted in many other cases.

Example 7. In [6, 7] approximate formulas are given for the local and the mean angular coefficient:

$$\varphi_\omega=\varphi^0\exp(-\alpha_\omega l_{\text{eff}}). \tag{10}$$

This expression is extremely simple. The effective length l_{eff} is determined easily if the surfaces are sufficiently far apart. If l_{eff} and the range of validity of Eq. (10) do not depend on the wave number, then there are no restrictions on its application to a real medium. Thus, according to the data in [7], two coefficients are available. We will write them down for $T = T_0$.

The mean angular coefficient for the end surfaces of a circular cylinder of diameter D and height H is

$$\varphi_4=(\sqrt{h^2+1}-h)^2\left[1-\varepsilon\left(\sqrt{H^2+D^2/4}\right)\right].$$

Here $h = H/D$. The formula yields satisfactory results when $h \geq 3$. When $h < 3$, the values of the coefficient are too high.

The local angular coefficient for a point on the lateral surface and the end surface of a cylinder at distance H is

$$\varphi_5=\left(\sqrt{h^2+1}-h-\frac{1}{2\sqrt{h^2+1}}\right)\left[1-\varepsilon\left(\sqrt{H^2+D^2/2}\right)\right].$$

The result is almost identical to that obtained by the exact formula, if $h \geq 3$. For $h < 3$ the values here are too low.

For parallel sides of a parallelepiped the approximate formula (10) was proposed in [6], with $l_{\text{eff}} = H(1 + \varphi^0)$. When $\alpha_\omega H \leq 0.1$, $\alpha_\omega \Delta < 1$, and $L/\Delta < 10$, the error is less than 3%. Unlike the coefficients φ_4 and φ_5, formula (10) is used here when the distance between the surfaces is very small. As $\alpha_\omega H \to 0$, this formula is preferable to the exact one on account of the singularity in the latter. Although l_{eff} does not depend on the wave number, the range of validity of the formula depends on it. For this reason, some further study is needed before the range of validity can be established for real-medium systems.

The representation of angular coefficients for real-medium systems in the form (10) has been accepted as universal in [11]. It is possible to apply here the concept of an angular coefficient also in the case of multiple reflections, without resorting to highly complex expressions.

In conclusion, we will consider two other independent methods of calculating the emissivity of three-dimensional bodies, of which one is well known whilst the other has been proposed earlier.

Mean Emissivity of a Convex Body. It is well known that the emissivities of various convex bodies do not differ much, if the dimension $l_0 = 4V/F$ is taken as the argument. Apparently, the maximum deviation is found in the case of a sphere and of an infinite plane strip. This deviation is 9% when $\alpha_\omega l_0 = 0.7$. For $\alpha_\omega l_0 > 4.6$ the deviation changes sign. In the case of a real body, there takes place an averaging of deviations over the spectrum. Therefore, overall emissivity deviates probably by less than 9%, the emissivity of a spherical shell being higher at almost any thickness encountered in practice (not for thicknesses

TABLE 1. Local Emissivity at the Center of the Plane Base of an Infinitely Long Gray Semicylinder (ε_2' is the local emissivity according to the exact formula (12); ε_2'' the emissivity according to the approximate formula (11); $\alpha_\omega R$ is the dimensionless optical radius)

$\alpha_\omega R$	ε_2'	ε_2''	$\alpha_\omega R$	ε_2'	ε_2''
0,05	0,0613	0,0633	1	0,697	0,699
0,1	0,118	0,121	2	0,902	0,901
0,5	0,457	0,461			

TABLE 2. Emissivity of an Infinitely Long Cylinder Containing Gaseous Carbon Dioxide at t = 1000°C (ε_3' is the emissivity according to Eq. (9) and the nomograms in [8]; ε_3'' the emissivity according to Eq. (11) with the Nevskii [9] values for the emissivity of a strip and a sphere; D is the diameter of the cylinder)

pD, m·atm	ε_3'	ε_3''	pD, m·atm	ε_3'	ε_3''
0,02	0,0556	0,0575	0,3	0,138	0,143
0,1	0,1007	0,1021	0,5	0,163	0,165

TABLE 3. Local Emissivity at the Center of the Plane Base of an Infinitely Long Semicylinder Containing Gaseous Carbon Dioxide at t = 600°C (ε_2' is the local emissivity according to Eq. (8) and the nomograms in [8]; ε_2'' the emissivity according to Eq. (11) with ε_1 taken from [9]; ε_0 is the emissivity calculated from the nomograms in [8]; R is the radius of the cylinder)

pR, m·atm	ε_2'	ε_2''	pR, m·atm	ε_2'	ε_2''
0,02	0,0722	0,0738	0,3	0,154	0,156
0,1	0,117	0,118	0,7	0,187	0,189

that are too large). A comparison with available data (Nevskii [9]) confirms this. There is a situation where the calculating procedure adopted for gray bodies is more accurate for real-medium bodies. The opposite is true in the following example.

Local Emissivity of a Symmetrical Convex Body. In Fig. 2 sections of two-dimensional bodies are shown. The sections perpendicular to them represent strips. Both parts of such a body are symmetrical with respect to the 00' axis, the line normal at point 0 of the hemisphere at which the local emissivity is to be determined. For the body generated by rotating the transverse cross section about the 00' axis we denote the emissivity at point 0 (of the sphere or hemisphere) by ε_0; the emissivity of a zone generated by rotation of the second section will be denoted by ε_1. For gray or monochromatic radiation we have

$$\frac{2}{\varepsilon} = \frac{1}{\varepsilon_0} + \frac{1}{\varepsilon_1}. \tag{11}$$

For the case in Fig. 2a, $\varepsilon \equiv \varepsilon_3$ is the emissivity of a cylinder. The maximum error is 1.44% when $\alpha_\omega D = 0.45$. For the case in Fig. 2b, $\varepsilon \equiv \varepsilon_2$ and $\varepsilon_0 = 1 - \exp(-\alpha_\omega R)$. According to the exact formula,

$$\varepsilon_2 = 1 - M(\alpha_\omega R). \tag{12}$$

The values of ε_2 obtained by the exact and by the approximate formula are given in Table 1. The deviation is somewhat greater than in Fig. 2a. For Fig. 2c, according to Eq. (11), we must write more generally

$$\frac{2\varepsilon_\infty}{\varepsilon} = \frac{\varepsilon_{0,\infty}}{\varepsilon_0} + \frac{1}{\varepsilon_1}. \tag{13}$$

Here $\varepsilon_\infty = \sin\beta$ is the emissivity for an infinitely continuous medium. Analogously, $\varepsilon_{0,\infty} = \sin^2\beta$. The values of ε_∞ and $\varepsilon_{0,\infty}$ are equal to the space angles subtending the bodies from the point 0. Since $\varepsilon_0 = \varepsilon_{0,\infty} \cdot [1 - \exp(-\alpha_\omega R)]$, $\varepsilon = \varepsilon_\infty \cdot \varepsilon_2$, from Eq. (13) we find Eq. (11) for a semicylinder with the deviations as listed in Table 1. When extending (13) and (11) to other configurations, one must remember that these are empirical formulas and have all the inherent shortcomings of such.

The use of Eqs. (11) and (13) for a real three-dimensional body arouses certain doubts, especially in the case of small thicknesses. Their error should be increasing in any case. Nevertheless, the values of ε_3 and ε_2 according to (11) for gaseous carbon dioxide have been compared with the respective values according to the formulas in Examples 3 and 2 (Tables 2 and 3). The deviations are almost entirely due to Eq. (11).

NOTATION

a_i, b_i, c_j, c_0	are coefficients independent of the wave number;
a_0	is the thermal absorptivity of a medium;
$a_0 = \varepsilon$ at $T = T_0$;	
a_{01}, a_{02}, a_{03}	are the absorptivity of a strip, a semicylinder, and a cylinder, respectively;
D, R	are the diameter and radius of a cylinder;
H	is the height of a cylinder or the distance between planes;
Δ, r, y	are distances in Fig. 1;
l	is the thickness of a plane-parallel strip, the path length of a ray;
l_{eff}	is the effective path length of a ray;
f_j	is an arbitrary function;
$I_{0\omega}$	is a Planck function, $cm \cdot W/m^2 \cdot sr$;
T, T_0	are the temperature of the medium and of the radiating surface, respectively, °K;
M, N_1, N_2, S_2	are intermediate Mikk functions;
E_3, Ki_n	are special functions [4, 5];
p	is the partial pressure of a component, atm;
$x = pl$, m·atm;	
$l_0 = 4V/F$;	
$\rho = H/R$;	
$h = H/D$;	
V	is the volume;
F	is the surface enveloping the volume;
m, u	are exponents;
α_ω	is the spectral absorptivity, m^{-1} (or $m \cdot atm)^{-1}$;
β	is an angle (Fig. 1);
ε_1, ε_2, ε_3, ε_0	are the emissivity of a strip, a semicylinder at the center of its plane base, a cylinder, and a solid of revolution, respectively;
φ_1, φ_2, φ_4, φ_5	are angular coefficients in a real-medium system considered in Examples 1, 2, and 7;
φ^0	is the angular coefficient for a diathermal medium;
ξ	is the local angular coefficient;
ω	is the wave number, cm^{-1};
$\tau_\omega = \alpha_\omega l$ or $\tau_\omega = \alpha_\omega x$;	
σT^4	is the thermal radiation density, W/m^2.

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